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Optimal Reconciliation of Seasonally Adjusted Disaggregates Taking Into Account the Difference Between Direct and Indirect Adjustment of the Aggregate

Francisco Corona[1], *Victor M. Guerrero*[2], *and Jesús López-Peréz*[1]

This article presents a new method to reconcile direct and indirect deseasonalized economic time series. The proposed technique uses a Combining Rule to merge, in an optimal manner, the directly deseasonalized aggregated series with its indirectly deseasonalized counterpart. The lastmentioned series is obtained by aggregating the seasonally adjusted disaggregates that compose the aggregated series. This procedure leads to adjusted disaggregates that verify Denton's movement preservation principle relative to the originally deseasonalized disaggregates. First, we use as preliminary estimates the directly deseasonalized economic time series obtained with the X-13ARIMA-SEATS program applied to all the disaggregation levels. Second, we contemporaneously reconcile the aforementioned seasonally adjusted disaggregates with its seasonally adjusted aggregate, using Vector Autoregressive models. Then, we evaluate the finite sample performance of our solution via a Monte Carlo experiment that considers six Data Generating Processes that may occur in practice, when users apply seasonal adjustment techniques. Finally, we present an empirical application to the Mexican Global Economic Indicator and its components. The results allow us to conclude that the suggested technique is appropriate to indirectly deseasonalize economic time series, mainly because we impose the movement preservation condition to the preliminary estimates produced by a reliable seasonal adjustment procedure.

Key words: Combining rule; contemporaneous restrictions; Monte Carlo experiment; vector autoregressive model; X-13ARIMA-SEATS.

1. Introduction

Seasonal adjustment of economic time series has been applied extensively by analysts at the U.S. Census Bureau since the 1950s with a method developed by themselves, called the "Census Method II". A variant of that method, the X-11, is at the core of X-13ARIMA-SEATS. In 1983, the Bank of Mexico carried out a systematic and formal seasonal adjustment project for economic time series (see Guerrero 1990, 1992), and seasonal adjustment has been implemented in the National Institute of Statistics and Geography, Mexico (INEGI) since the 1990s according to procedures adopted by many official

[1] Instituto Nacional de Estadística y Geografía (INEGI), Research, Avenida Patriotismo 711, Torre A, CP 03730, CDMX, México, Mexico. Emails: franciscoj.corona@inegi.org.mx, and jesus.lopezp@inegi.org.mx
[2] Instituto Tecnológico Autónomo de México (ITAM), Department of Statistics, Rio Hondo 1, Progreso Tizapán, CP 01080, CDMX, Mexico. Email: guerrero@itam.mx
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statistics agencies around the world. Nowadays, official seasonal adjustment of economic time series is carried out by INEGI using the X-13ARIMA-SEATS approach. Additionally, for some relevant economic time series, such as the Gross Domestic Product (GDP), the seasonal adjustment is carried out with the consensus of the Bank of Mexico, the Ministry of Finance, and the Ministry of Economy.

The goal of seasonally adjusting time series is to obtain information free of seasonal patterns, since these are not related to economic events or to government policies. That is why seasonal adjustment is relevant to better determine the state of the economy and, consequently, for policy making.

We can seasonally adjust a time series composed of several disaggregated time series relatively easily, by directly applying the seasonal adjustment program to the aggregate. Alternatively, we can use an indirect approach by applying the seasonal adjustment to each individual series and aggregating the adjusted time series to obtain the adjusted aggregate. However, the resulting adjusted series obtained indirectly will usually differ from the one obtained directly.

In Mexico, official seasonal adjustment of an aggregated time series is carried out directly. However, some users of seasonally adjusted figures consider the discrepancies between a seasonally adjusted aggregate obtained indirectly with that obtained by a direct method basically as a mistake. Therefore, the discrepancies between the two approaches are considered unacceptable by those people, because they are difficult to interpret. However, those users overlook the adequacy of the seasonal adjustment procedure employed, because they tend to think that deseasonalization is essentially a linear procedure that can be carried out by just clicking a button. If that were the case, the adjustment results may lead in many situations to wrong conclusions.

There is a tradeoff between "structural" interpretation of the seasonal adjustment and its statistical optimality. With the direct approach, we can verify the statistical properties of the aggregated seasonally adjusted series, but the aggregation of the individual seasonally adjusted components do not produce the directly adjusted aggregate. On the other hand, the indirect approach produces an adequate structural explanation, but the statistical properties of the seasonally adjusted aggregated series are not guaranteed. INEGI applies the direct approach for seasonal adjustment, in order to ensure that all the time series components are correctly deseasonalized.

A typical concern that arises with the direct approach is the desynchronizing of movements between the aggregate and its components. For example, the adjusted Global Economic Activity Indicator of Mexico (IGAE in Spanish), the monthly proxy of GDP, decreased in September 2006, but its three adjusted Grand Economic Activities (GA) increased. In contrast, in July 2007 the adjusted components of IGAE decreased, but IGAE as a whole increased. Therefore, it is necessary to use a statistical procedure to reconcile these results.

In this work, we refer to the statistical properties and optimality of the methods employed. By this, we mean that we have to bear in mind that the seasonal adjustment methodology arises from statistical models and assumptions leading to optimal procedures, in terms of formal statistical criteria. Therefore, we should be able to identify the assumptions underlying the methodology in order not to apply it as a cookbook recipe, but as a formal statistical method. Once we do that, we need to validate the assumptions with the data at hand, so that the results produced by the methodology become empirically valid. This, in turn, guarantees the optimality and reliability of the results.

Some authors have proposed to solve this problem by using benchmarking and statistical models. For example, Den Butter and Fase (1991) use contemporaneous constraints, Di Fonzo and Marini (2005) incorporate jointly contemporaneous and temporal constraints. Additionally, Stuckey et al. (2004), Quenneville and Rancourt (2005), Dagum and Cholette (2006) and Quenneville and Fortier (2012) propose to use two-step procedures, while McElroy (2017) uses multivariate techniques to deseasonalize economic time series. However, this problem still remains as observed by Guerrero et al. (2018).

In this article, we propose to use a particular case of the Combining Rule (CR) presented in Guerrero and Nieto (1999) to multivariate time series in order to reconcile time series previously deseasonalized with the X-13ARIMA-SEATS program, maintaining the statistical properties of the direct approach, but restricting the linear aggregation of the components to equal its seasonally adjusted aggregate.

This reconciliation approach is new, since we carry out this task by means of a CR that provides optimal results, in the sense that the resulting deseasonalized series of disaggregates is as close as possible to the directly deseasonalized disaggregates, subject to the information provided by the directly seasonally adjusted aggregate. We emphasize that this CR provides optimal results only when the assumptions underlying the statistical model involved are valid, which have to be verified with the data at hand. Only when the data provide the required empirical support for the model, can we say that the results obtained with the CR are optimal.

This article is organized as follows. Section 2 provides a brief summary of the X-13ARIMA-SEATS program and discusses some features of the direct and indirect methods. Section 3 explains the CR in order to reconcile the deseasonalized time series obtained with the direct approach. Section 4 describes the Monte Carlo experiment and shows its results. Section 5 presents an empirical application to the components of IGAE. Finally, Section 6 concludes.

2. A Brief Summary of X-13ARIMA-SEATS Seasonal Adjustment: Direct and Indirect Approaches, and Reconciliation

We assume the classical "additive" time series decomposition for $t = 1, \ldots, T$

$$Y_t = T_t + S_t + I_t \tag{1}$$

where Y_t is the observed time series, T_t is the trendcycle component, S_t is the seasonal component and I_t is the irregular component. For example, T_t contains the longrun movement of the time series plus fluctuations related to economic phenomena, S_t contains the systematic patterns that occur in the corresponding time period (month or quarter) and I_t, the unpredictable component that incorporates, for example, outliers and/or random effects.

Alternatively, the previous equation can be expressed as the "multiplicative" representation

$$Y_t = T_t \times S_t \times I_t. \tag{2}$$

If we apply logarithmns to this representation, we can express this model again as additive. For hybrid representations, say additive-multiplicative, see for instance, Findley et al. (1998) and Ladiray and Quenneville (2012).

The essence of X-13ARIMA-SEATS lies in extracting the components T_t, S_t and I_t using a variety of statistical methods. We emphasize the following two steps:

1. ARIMA regression: This step is useful to remove calendar effects, such as those of trading days, leap year, Easter, outliers and structural breaks, among many others. For this objective, we have to estimate a seasonal ARIMA model with exogenous variables. This model is also used to generate backcasts and forecasts, which are necessary to apply the seasonal filters in the next step.
2. Seasonal filters: The X-13ARIMA-SEATS program is based on the original X-11 sequence of predefined moving average filters, applied as an iterative process. After the series is preadjusted and extended with backcasts and forecasts (as indicated in the previous step), it goes through three rounds of filtering and extreme value adjustments called: Initial Estimates, Final Seasonal-Irregular Ratios and Final Components. The goal is to obtain Final Seasonal Factors, Final Seasonally Adjusted Series, Final Trend Cycle and Final Irregular series.

Specifically, these two steps require a descriptive analysis, as well as to determine the order of integration and the type of transformation to be applied to the time series. We then need to estimate a Regression model with ARIMA errors (RegARIMA) (see Lytras et al. 2007), to select and apply the seasonal filters, to evaluate the assumptions of the RegARIMA model and to verify that the adjusted time series does not show seasonal patterns. Finally, we have to decide whether or not to apply the indirect method to the aggregates. Alternatively, the TRAMO-SEATS (Gomez and Maravall, 1996) approach is based on the estimation of ARIMA models from which we can extract the signals that allow us to estimate the seasonal factors of the time series.

As we said before, in the direct method the deseasonalized time series, $y_{1t}, \ldots, y_{Nt}$ and the aggregate, Y_t, are adjusted without considering any hierarchical structure. In the indirect approach we consider specifically the case $Y_t = \sum_{i=1}^{N} y_{it}$, so that the seasonally adjusted aggregate is obtained as the sum of the components. In the basic indirect approach it is important to consider that: (1) The disaggregates are directly deseasonalized, consequently, these time series can be deseasonalized applying statistical criteria. (2) The sum of the components generates the aggregate. Hence, the interpretation of Y_t is very simple. (3) The seasonal factors of Y_t are not considered explicitly. And (4) consequently, the seasonal factors of Y_t are not estimated. Furthermore, we emphasize that the statistical quality of the seasonal adjustment of Y_t cannot be evaluated.

The first two points are in favor of the indirect method, while the rest support the direct approach. Given this disjunctive, Eurostat recommends (Eurostat 2015, 34) that the best way to proceed, from a statistical point of view, is to apply the direct method. Alternatively, it is reasonable to apply benchmarking to the disaggregates in order to satisfy contemporaneous restrictions and to verify that there is no residual seasonality. Finally, it is necessary to have proper justification for applying any of the two approaches. Frequently, the former argument is the most important to decide which method should be selected and INEGI follows this recommendation.

The problem of ensuring aggregation in a system of time series is commonly referred in the literature as benchmarking to contemporaneous or temporal constraints. This way, the reconciliation of systems of time series subjected to constraints is very common in official statistics. The restrictions typically arise from national accounts systems, for example to keep duality in the GDP accounts of uses and resources. Such accounting restrictions arise from two main sources: first, contemporaneous constraints, so that some linear combinations of the variables have to be fulfilled every observation period, and second, temporal constraints, when the restrictions come from low frequency series whose high-frequency counterparts must be in line with. Another important source of discrepancies, on which this work is focussed, are those that arise from the seasonal adjustment process, where benchmarking is used to restore the additivity in systems of directly deseasonalized component series, in order to be in line with both the seasonally adjusted marginal aggregates and the grand-total series.

The statistical procedures to realign a set of data in order to satisfy a set of accounting restrictions is known in general as balancing or reconciliation for broad surveys in the area (see Dagum and Cholette 2006; Pavía-Miralles 2010; Chen 2012; Infante 2017). In those studies, the techniques are classified as balancing (to adjust variables), benchmarking of time series (to adjust variables in the time dimension) and reconciliation (when the two constraints are to be met), the latter is needed because in most cases the balancing techniques do not preserve the dynamics of the related indicator. Reconciliation techniques are divided into simultaneous and two-step approaches. The former includes the works of Fernández (1981), Rossi (1982) and Di Fonzo (1990). However, when the system of time series is very large so that a simultaneous solution is computationally intensive, the two-step approach is necessary.

In particular, two-step reconciliation of direct seasonally adjusted series to meet accounting restrictions have been studied by Di Fonzo and Marini (2005), Quenneville and Rancourt (2005), Di Fonzo and Marini (2011), Quenneville and Fortier (2012), Di Fonzo and Marini (2015), among many others. For example, Quenneville and Rancourt (2005) restore the additivity of a system of deseasonalized time series in such a way that their sum agrees with that derived from the totals. This is done by first applying additivity benchmarking to every deseasonalized series and then a balancing procedure is applied to the seasonal component series. More recent developments include Proietti (2011) and McElroy (2018); the latter presents a solution to the phenomenon called crossaggregation that is, the possibility that the aggregation of many series deemed to be nonseasonal may exhibit seasonality.

Another statistical perspective of combining information can be applied to the problem of data reconciliation. In that sense, Guerrero and Nieto (1999) and Guerrero and Peña (2000, 2003) addressed the problem of combining information and derived an optimal CR for both univariate and multivariate problems, respectively. The CR provides a unified framework to attack time series problems, including forecast updating, missing data, restricted forecasting, temporal disaggregation, outliers and structural changes, and contemporaneous disaggregation. Particular cases of the CR include the solutions derived by Lisman and Sandee (1964), Boot et al. (1967), Denton (1971) and Chow and Lin (1971), among others.

Fernáandez (1981) describes a temporal disaggregation technique working on a single time series, as Lisman and Sandee (1964), Boot et al. (1967), Ginsburgh (1973) and Litterman (1983) do. Rossi (1982) and Di Fonzo (1990) instead, deal with temporal disaggregation of a system of time series. In addition, Rossi (1982) presents a two-step adjustment procedure, Fernández (1981) – univariate – and Di Fonzo (1990) – multivariate – which are natural extensions of Chow and Lin (1971).

The solution presented here uses the approach of Guerrero and Nieto (1999) to address the additivity problem of a system of deseasonalized time series. We consider that this approach makes efficient use of the available information by taking into consideration the relations among variables through the covariance matrix, a typical problem faced in National Statistics Agencies. Nonetheless, there are some recent advances in the literature on two-step reconciliation, for example, a post-adjustment correction technique that is less used in practice. In particular, those techniques are employed in statistical agencies that have experts in the field (Infante 2017).

3. Combining Rule to Reconcile Deseasonalized Economic Time Series

The basic idea is to reconcile a deseasonalized time series vector using preliminary estimates obtained with another method, in this case the time series directly deseasonalized with the X-13ARIMA-SEATS program. Thus, we use multivariate time series linear models to reconcile the preliminary estimates with respect to their aggregate, verifying movement preservation between reconciliated time series and preliminary estimates. The proposed reconciliation procedure is optimal from a statistical point of view and is based on Vector Autoregressive (VAR) models. For this purpose, we consider the CR of Guerrero and Peña (2000, 2003) to contemporaneously reconcile preliminary estimates to an aggregate. Specifically, we consider the CR for multivariate time series derived by Guerrero and Nieto (1999) where preliminary time series are subjected to some contemporaneous restrictions.

Consequently, we define the vector to be reconciled as $\mathbf{Z} = (\mathbf{Z}'_1, \ldots, \mathbf{Z}'_T)'$ where $\mathbf{Z}_t$ is of dimension k. Then, assume that $\mathbf{W}_t$ and $\mathbf{W}$ are preliminary estimates of $\mathbf{Z}_t$ and $\mathbf{Z}$ so that,

$$\mathbf{Z}_t = \mathbf{W}_t + \mathbf{S}_t, \tag{3}$$

where $\mathbf{S}_t$ admits a stationary VAR representation of order $p \geq 1$, so that

$$\Pi(B)\mathbf{S}_t = \mathbf{a}_t, \tag{4}$$

with $\Pi(B)$ a polynomial matrix in the backshift operator B and $\{\mathbf{a}_t\}$ a sequence of zero mean White Noise random vectors. If we let $\mathbf{S}$ be a vector defined in the same fashion as $\mathbf{Z}$ we can write

$$\Pi\mathbf{S} = \mathbf{a}, \tag{5}$$

where $E(\mathbf{a}) = \mathbf{0}$ and $E(\mathbf{aa}') = I_T \otimes \Sigma$ with I_T the identity matrix, $\Sigma = E(\mathbf{a}_t \mathbf{a}'_t)$ and $\otimes$ denoting Kronecker product.

The matrix Π has the coefficients $\pi_1, \ldots, \pi_p$ of the polynomial $\Pi(B)$ that is,

$$\Pi = \begin{pmatrix} I_k & 0 & 0 & \cdots & 0 & 0 \\ -\pi_1 & I_k & 0 & \cdots & 0 & 0 \\ \cdots & \vdots & \cdots & \ddots & \cdots & \cdots \\ -\pi_p & -\pi_{p-1} & -\pi_{p-2} & \cdots & 0 & 0 \\ \cdots & \vdots & \cdots & \ddots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -\pi_1 & I_k \end{pmatrix}. \tag{6}$$

Then, given $\mathbf{W}$, we assume the representation $\Pi\mathbf{S} = \mathbf{a}$ also holds true, with

$$E(\mathrm{a}|\mathbf{W}) = 0 \text{ and } E(\mathbf{aa}'|\mathbf{W}) = P \otimes \Sigma, \tag{7}$$

where P is a positive definite matrix, deduced from the data. We propose to use the same VAR polynomial for both $\mathbf{Z}_t$ and $\mathbf{W}_t$ which amounts to saying, in the present situation, that both final and preliminary deseasonalized series share the same autocorrelation structure. This is because we require a VAR model that enables us to capture the empirical regularities in the data. In no way are we supposing that it is the right model. We do this in order to avoid a time series identification problem.

Since the preliminary series $\{\mathbf{W}_t\}$ and that to be estimated $\{\mathbf{Z}_t\}$, admit the same VAR(p) representation we have

$$\Pi(B)\mathbf{Z}_t = \mathbf{D}_t + \mathbf{a}_{\mathrm{Z},t} \text{ and } \Pi(B)\mathbf{W}_t = \mathbf{D}_t + \mathbf{a}_{\mathrm{W},t}, \tag{8}$$

where $\{\mathbf{a}_{\mathrm{Z},t}\}$ and $\{\mathbf{a}_{\mathrm{W},t}\}$ are White Noise processes, with covariance matrices $\Sigma_Z \neq \Sigma_W$ and with the vector $\mathbf{D}_t$ that contains deterministic elements. We also assume that there exists a $k-$dimensional vector $\mathbf{Y}$, containing some additional information about the vector to be estimated. In our case, $\mathbf{Y}$ contains the aggregation of $\mathbf{Z}$ so that the contemporaneous restrictions are given by

$$\mathrm{Y}_t = \mathbf{c}'\mathbf{Z}_t \text{ for } t = 1, \ldots T, \tag{9}$$

with $\mathbf{c} \neq \mathbf{0}$ a known constant $k-$dimensional vector, for instance $\mathbf{c}$ is a vector of $1s$ when the aggregation corresponds to a sum, or $\mathbf{c}$ has $1/k$ in all its entries when the aggregation is given by an average.

Finally, Guerrero and Nieto (1999) show that the estimator of $\mathbf{Z}$, given $\mathbf{W}$ and $\mathbf{Y}$, is obtained from the CR as

$$\hat{\mathbf{Z}} = \mathbf{W} + A(\mathbf{Y} - C\mathbf{W}), \tag{10}$$

with

$$Cov(\hat{\mathbf{Z}} - \mathbf{Z}|\mathbf{W}) = (I_{kT} - AC)\Psi, \tag{11}$$

where

$$A = \Psi C'(C\Psi C')^{+} \text{ and } \Psi = \Pi^{-1}(P \otimes \Sigma)\Pi^{-1'} \tag{12}$$

where C is a known matrix used to represent the contemporaneous restrictions and the superindex $+$ denotes Moore-Penrose generalized inverse.

The empirical strategy to obtain $\hat{\mathbf{Z}}$, as proposed by Guerrero and Nieto (1999) is as follows:

1. Use the direct approach suggested by X-13ARIMA-SEATS to obtain the deseasonalized time series $\mathbf{W}_t$ of the disaggregates of $\mathbf{Y}_t$, consequently $\mathbf{W}_t$ contains the preliminary estimates of $\mathbf{Z}_t$.

2. Estimate a VAR(p) model for $\mathbf{W}_t$ using Ordinary Least Squares (OLS), which amounts to using $P = I$ as a tentative assumption. By using OLS we allow for the presence of unit roots and cointegration, which might be present since seasonally adjusted data is typically at least $I(1)$. To carry out this step, identify the VAR model for the disaggregates in levels. Then, choose the number of lags by either an automatic procedure, namely by minimizing the AIC, BIC or HannanQuinn criteria, or by means of the likelihood ratio procedure. For more information on this issue see Lütkepohl (2005).

3. Use the estimates of the VAR(p) to compute $\hat{A} = \hat{\Psi}C'(C\hat{\Psi}C')^{+}$ where $\hat{\Psi} = \hat{\Pi}^{-1}\hat{\Sigma}\hat{\Pi}^{-1'}$. Then, calculate $\hat{\mathbf{Z}}_t$, as well as the estimated variance $\hat{\Sigma}$ with the usual unbiased expression provided by OLS.

4. Compute the discrepancies $\mathbf{D}_t^* = \hat{\mathbf{Z}}_t - \mathbf{W}_t$.

5. Test if $\mathbf{D}_t^*$ is multivariate uncorrelated. We use the Portmanteau and the Breusch-Godfrey tests for serially correlated errors to this end.

6. If the null hypothesis is not rejected, finish the procedure. Otherwise, assume that $\Lambda\mathbf{D}^* = (Q \otimes I)\Pi\mathbf{D}^* = \mathbf{u}$ where Q is a non-singular matrix such that $QPQ' = I$. In this case note that

$$E(\mathbf{u}\mathbf{u}'|W) = (Q \otimes I)(P \otimes \Sigma)(Q' \otimes I) = I \otimes \Sigma, \tag{13}$$

and

$$\Psi = \Pi^{-1}(P \otimes \Sigma)\Pi^{-1'} = \Lambda^{-1}(I \otimes \Sigma)\Lambda^{-1'}. \tag{14}$$

Consequently, repeat steps 2 and 3 to $\mathbf{D}_t^*$ using the OLS estimates to obtain $\tilde{\mathbf{Z}}_t$ as well as $\hat{\Sigma}$ which now includes the autocorrelation structure, that is, we propose to use a feasible version of Generalized Least Squares (GLS) to guarantee the optimality of the results according to Guerrero and Nieto (1999).

7. In order to empirically verify that the final vector of discrepancies $\mathbf{D}_t^* = \tilde{\mathbf{Z}}_t - \mathbf{W}_t$ is unbiased (zero mean) and stationary, that is, movement preservation is achieved, Guerrero and Corona (2018) propose to use a statistical method to prove this hypothesis.

It is important to comment that the term $A(\mathbf{Y} - C\mathbf{W})$ added to the seasonally adjusted disaggregate time series, see Equation (10), may distort the optimality of the seasonal adjustment by adding some seasonality back in. We are aware of this undesired effect, but it requires further investigation, for example following McElroy's (2018) ideas. Thus, as a future work, we should design a diagnostic procedure that enables an analyst to check for this situation and suggest a remedy, but our current proposal does not consider this type of situation in detail. Also, note that GLS allows for the possibility of such error by having a variance-covariance matrix different from the identity. Our procedure is akin to Cochrane-Orcutt's since it allows us to incorporate the error autocorrelation in the parameter estimation, letting the resulting VAR polynomial for the error to be decided by the data themselves.

When the elements of $\mathbf{W}_t$ are co-integrated, the VAR model in levels is equivalent to its respective Vector Error Correction (VEC) representation. Consequently, the procedure previously described is valid for some types of non-stationarity in the preliminary time series, and stationary transformations are not considered appropriate when estimating the VAR model for $\mathbf{W}_t$. Also, if $\mathbf{D}_t^*$ is stationary, that is, the movement preservation is achieved, the estimate of $\tilde{\mathbf{Z}}_t$ and $\mathbf{W}_t$ are cointegrated. If $\mathbf{W}_t$ are non-stationary and non-cointegrated, the proposed procedure performs as a benchmarking method and its sample performance should be studied, particularly by way of a Monte Carlo analysis.

Additionally, Guerrero and Nieto (1999) show that we can include deterministic regressors in the VAR model. Moreover, note that if the $k-$dimensional vector $\mathbf{c}$ contains the term $1/s$, where s is the number of periods of the year (for example, $s = 12$ for monthly data), the restrictions are temporal. This is equivalent to the force procedure in X-13ARIMA-SEATS.

It is convenient to emphasize the distinction between the extension of Denton's procedure considered by Di Fonzo and Marini (2005) and the method proposed in this article. The former procedure focuses on the movement preservation principle, which is highly desirable. However, it does not indicate how to validate the assumptions of the underlying statistical model that leads to the benchmarking formulas. On the contrary, the work of Guerrero and Nieto (1999) seeks to accomplish both tasks; firstly, by paying attention to the empirical validation of the VAR model with the available data, and then by taking into account the aggregated series as a set of restrictions to be fulfilled. This is what we schematically presented in the empirical strategy of this article.

4. Monte Carlo Experiment

4.1. Seasonal Dynamic Factor Model

In order to evaluate the finite sample performance of the suggested reconciliation procedure, we carried out a Monte Carlo experiment where the time series are simulated using Seasonal Dynamic Factor Models (SDFM). We propose the use of SDFM since we wish to simulate different situations that can occur in practice, mainly when INEGI deals with time series subjected to a seasonal adjustment process. The idea of the SDFM is that the observations, $\mathbf{Z}_t$, are generated by common movements plus individual seasonal patterns. The SDFM is expressed as follows

$$\begin{aligned} \mathbf{Z}_t &= \lambda \mathbf{F}_t + \boldsymbol{\varepsilon}_t, \\ (I - \Phi B)\mathbf{F}_t &= \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{t} + \boldsymbol{\eta}_t, \qquad (15) \\ (I - \Omega B^s)(I - \Gamma B)\varepsilon_t &= \mathbf{u}_t, \end{aligned}$$

where λ is the loading matrix of dimension $k \times r\,(r < k)$, that indicates the contribution of the common factors, $\mathbf{F}_t$, on the observations. The common factors, their disturbances, $\boldsymbol{\eta}_t$, and the constant vectors, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, are of dimension r and the linear trend is denoted as $\mathbf{t}$. On the other hand, the idiosyncratic component, $\boldsymbol{\varepsilon}_t$, and the errors, $\boldsymbol{u}_t$, are $k-$ dimensional random vectors. The diagonal coefficient matrices are Φ, Γ and Ω. Finally, we assume the covariance matrices Σ_η and Σ_u are positive-definite.

We should note several econometric implications of this SDFM. First, the seasonal patterns are specific for each time series, that is, there are not dynamic dependences among the time series. Second, the stochastic nature of the time series depends on both the stationarity of the common factors and the idiosyncratic component. If both $\boldsymbol{F}_t$ and $\boldsymbol{\varepsilon}_t$ are integrated of order zero, the observed time series are stationary. If $\boldsymbol{F}_t$ is integrated of order one and $\boldsymbol{\varepsilon}_t$ is stationary, $\boldsymbol{Z}_t$ are cointegrated, being $\boldsymbol{F}_t$ the common trends. If $\boldsymbol{F}_t \sim I(0)$ and $\boldsymbol{\varepsilon}_t \sim I(1)$, $\boldsymbol{Z}_t$ are non-stationary but not cointegrated. Finally, if both $\boldsymbol{F}_t$ and $\boldsymbol{\varepsilon}_t$ are $I(1)$, $\boldsymbol{Z}_t$ are also random walks.

It is interesting to realize that when the variance of the common factors is much larger than the variance of the idiosyncratic component, the seasonal pattern will only have a small impact on the observations. Thus, in order to avoid this situation, the configuration of the parameters in the Monte Carlo analysis considers serial correlation in the idiosyncratic components and weak cross-correlation.

4.2. *Configuration*

The configuration of the SDFM was discussed with analysts working at INEGI's Directorate of Econometric Studies in order to simulate different time series that cover several situations that the analysts usually deal with in practice. This way, we consider six Data Generating Process (DGP) and the number of monthly ($s = 12$) time series are $k = 4$, hence, we take into account three disaggregates and one aggregate with a time span of $T = 200$ observations. We consider $R = 500$ replicates for each DGP, also called model. In our simulations Z_1 is obtained as the sum of the other three time series generated by the SDFM.

The loadings matrix is simulated as $\lambda \sim N\ (0,\ 1)$ for each replicate, maintaining the loading weights fixed for each DGP, $\Sigma_\eta = diag(1)$, $\boldsymbol{\alpha} = 50$ and a seasonal behavior given by $\Omega = diag\ (U \sim (-1, 1))$. Consequently, we have R different seasonal patterns. Finally, we allow for weak cross-sectional correlated idiosyncratic errors by simulating a covariance matrix as follows

$$\sum_u = \begin{pmatrix} 6.94 & 9.80 & 8.74 \\ 9.80 & 13.91 & 11.94 \\ 8.74 & 11.94 & 13.53 \end{pmatrix}$$

Following Bai and Ng (2002), among many others, we define Σ_u as such to allow for cross section dependence in the idiosyncratic components so that the model has an approximate factor structure. It is more general than a strict factor model, which assumes no correlation, so that the results hold also for strict factor models. Specifically, we simulate this matrix as $\Sigma_u = \Xi'\Xi$ where the elements of Ξ are simulated as $U \sim (1, 2)$.

The first model (M1) considers $r = 1$ common factor with $\Phi = 0.5$ and $\beta = 0.1$ and the idiosyncratic component is White Noise. The second model (M2) is simulated as the M1 model but with $\Gamma = diag(0.5)$. Consequently, in this model the seasonal idiosyncratic errors are also serially correlated. The third model (M3) is as the M1 model, but with $\Phi = 1$ and $\beta = 0$. Hence, the pairs of elements of $\mathbf{Z}_t$ are cointegrated. The fourth model (M4) is a hybrid model with two common factors, one non-stationary and another stationary, $\Phi = (1,\ 0.5)$ and $\Gamma = diag(0.25)$. The fifth model (M5) considers one stationary common factor, $\Phi = (0.5)$ and $\Gamma = diag(1)$, that is, in this model the time series are non-stationary and non-cointegrated. Finally, the sixth model (M6) is as the M5 model but with $\Phi = 1$, in other words, the time series are pure random walks. In sum, the main idea is to generate different stationary and non-stationary time series, with serially correlated errors or non-stationary and weak cross-sectionally correlated elements with different seasonal patterns among them.

Carrying out good preliminary estimation is very important to attain accurate final estimates with similar statistical properties. Thus, we compute the F-test on seasonal dummies proposed by Lytras et al. (2007) to check for the presence of deterministic seasonality. This test is based on the estimates of the regression dummy variables and the corresponding t-statistics of the RegARIMA model, by computing

$$F^M = \frac{\hat{\chi}}{11} \times \frac{T-d-h}{T-d}$$

where $\hat{\chi} = \hat{\beta}' \ [var\ (\hat{\beta})]^{-1}\ \hat{\beta}$, with $\hat{\beta}$ the RegARIMA coefficients, d is the number of stationary differences and h the number of estimated parameters in the RegARIMA model. Note that the null hypothesis indicates that all the parameters are simultaneously equal to zero, which denotes the absence of seasonality. In each replicate, we adjust a RegARIMA model with seasonal dummy variables, where the ARIMA parameters are automatically detected by using the procedure of Hyndman and Khandakar (2008)

In order to empirically verify movement preservation, we verify that $\mathbf{D}_t^*$ has zero mean, with both a t-test and an ADF test for the discrepancies to be stationary. Furthermore, it is worth mentioning that INEGI's seasonal adjustment personnel carefully deseasonalize one series at a time, without resorting to an automatic model with the X-13ARIMA-SEATS program. They assess every step when applying the program: order of integration, type of transformation, ACF and PACF analysis of time series, selection of exogenous variables, analysis of residuals of the RegARIMA model, study of the seasonal factors, and so on.

As noted in the Introduction, a motivation of this article is to produce deseasonalized time series that can appropriately explain the movements of the aggregates with those of the disaggregates. The direct method does not guarantee that result, contrary to the reconciled indirect method proposed in this work. Therefore, for the preliminary time series, we decided to compute in each replicate the positive fails, $PT = \sum_{t=1}^{T} P_t$ and the negative fails, $NT = \sum_{t=1}^{T} N_t$ where

$$P_t = (1|\nabla Y_t > 0, \nabla \mathbf{W}_t < \mathbf{0}), \tag{16}$$

and

$$N_t = (1|\nabla Y_t < 0, \nabla \mathbf{W}_t > \mathbf{0}) \tag{17}$$

where ∇ operator, defined as a function of the backshift operator B, as $\nabla = 1 - B$. These measures count the number of "fails", in a structural sense, by using the direct method for the seasonal adjustment.

4.3. Results

To verify that the preliminary estimates used by automatic X-13ARIMA-SEATS, correctly deseasonalize time series, Figure 1 shows box-plots of the p-values for the F^M tests obtained when applying the direct and automatic method for each time series and the six models (M1 to M6).

The F^M test results indicate that in all cases, the residual time series are free of seasonal patterns across series and models. Consequently, the preliminary estimates constitute an appropriate input to apply our suggested approach. To empirically evaluate movement preservation, we introduce the following MP statistic, which we calculate for each model

$$MP = \sum_{r=1}^{R} MP_r / R.$$

with

$$MP_r = (1 | p_{mean,r} > \alpha, p_{ADF,r} < \alpha)$$

where $p_{mean,r}$ is the p-value of the t test for mean difference between $\hat{\mathbf{Z}}_t$ and $\mathbf{W}_t$, $p_{ADF,r}$ is the p-value of the ADF test applied to $\mathbf{D}_t^*$ and α is the significance level. This statistic indicates the percentage of times that $\mathbf{D}_t^*$ is jointly unbiased and stationary. The results are summarized in Table 1.

Note that, in this case, movement preservation is more frequently achieved for stationary or cointegrated models. It is reasonable to expect that once the seasonal pattern

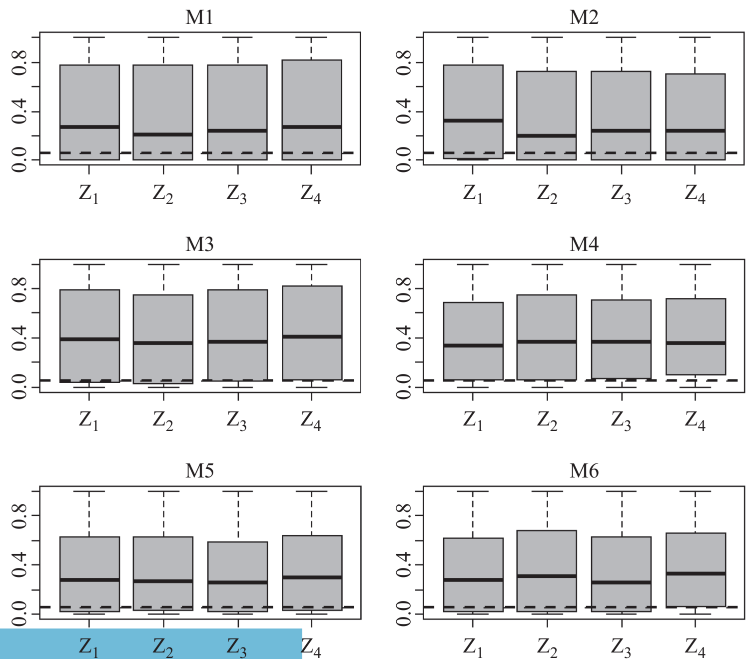

Fig. 1. Z_1 is the sum of the other three series. The p-values F^M tests obtained when applying the direct method for each time series and model. The dotted line indicates the significance value $\alpha = 0.05$.

Table 1. MP statistic with $\alpha = 0.05$.

Model	MP
M1	0.97
M2	0.97
M3	0.98
M4	0.99
M5	0.88
M6	0.88

is removed from $\mathbf{Z}_t$, when there are common trends any transformation of them will remain cointegrated.

However, for all models the movement preservation is frequently achieved. Otherwise, although the restrictions are satisfied, we cannot empirically verify the assumption that validates the procedure, where $\mathbf{W}_t$ and $\mathbf{Z}_t$ follow the same VAR(p) model. Figure 2 shows graphs of the empirical performance of sign changes between at least one disaggregate with its respective aggregate, for the six models. For each model, we present two histograms, one for positive fails calculated as indicated by Equation (16) and the other for negative fails, calculated as in Equation (17). In each graph, the horizontal axis has the number of fails, while the vertical axis shows the observed frequencies for each interval of fails.

We can see that models M1 and M3, and its hybrid M4, exhibit a smaller number of fails very often, and a larger number of fails rarely with some peaks. Let us recall that model M4 is a hybrid of M1 and M3, in which the time series are cointegrated, unless $\lambda_1 = 0$. These three models are slightly more complicated than models M2, M5, and M6, and thus more regularly show sign changes with a smooth decay in the number of fails.

The main conclusion of these results is related to the fact that, when the time series have common movements in the short and/or long run, and the individual component is weakly serially correlated, the synchronization among the disaggregates and its aggregate tends to have more fails. Note that we have a total of $R \times T = 100,000$ possibilities of fails in each model, consequently, the maximum percentage is around 1.4%. Then, when we use the

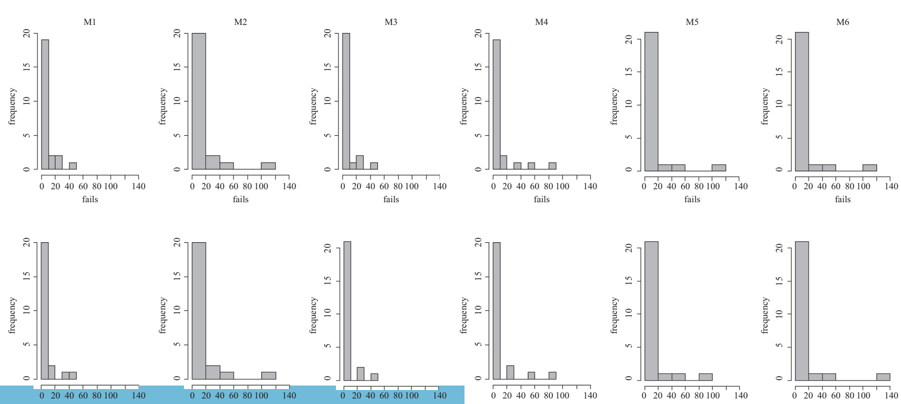

Fig. 2. Frequency of the number of fails due to applying direct seasonal adjustment. Top panel corresponds to P_t and bottom panel indicates N_t.

direct method, the percentage of inconsistencies tends to be small and when we restrict the preliminary estimates with contemporaneous restrictions, the inconsistencies disappear.

5. Empirical Application

The empirical application was carried out with IGAE and its three Grand Economic Activities (GA), to verify that movement preservation of the preliminary time series are satisfied, as well as the structural explanation of the monthly variations. Additionally, to provide a basis for comparison we applied the procedure suggested by Stuckey et al. (2004), denoting the reconciled estimates by this procedure as $\hat{\mathbf{W}}_t$.

We first define the vector of linear restrictions $\mathbf{c} = (0.032,\ 0.342,\ 0.627)$, which we obtain by recursively solving a system of simultaneous equations with the original series. We refer to the components of IGAE as GA1 for the primary activities, GA2 to the industrial activities and GA3 to the services, according to the North American Industrial Classification System 2018.

Here, we follow what is nowadays the standard practice of VAR modelling. We recommend the interested reader to refer to Lütkepohl (2005) for details on this topic. The deterministic specification of the VAR model considers both constant and linear trend. We select the length of the VAR model according to the Schwarz criterion and for $\mathbf{W}_t$ we obtain as optimum number of lags, $p = 1$. The Johansen test of cointegration indicates two cointegration relationships, hence, the VEC and VAR models are equivalent. Therefore, we estimate the parameters to obtain $\hat{\mathbf{Z}}_t$ and by applying the Portmanteau and the Breusch-Godfrey test for $\mathbf{D}^*_{t,}$ we conclude that it is autocorrelated with p-value of 0.0.

Consequently, we estimate a VAR model by GLS without constant nor trend to $\mathbf{D}^*_t$, with $p = 12$ chosen with Schwarz criterion. Now, we obtain $\tilde{\mathbf{Z}}_t$ and to verify the unbiasedness of the new vector of discrepancies, we apply a t-test for means obtaining p-values of 0.99 for the three cases. Additionally, the ADF-tests indicate that the components of $\mathbf{D}^*_t$ are stationary, providing evidence that movement preservation is achieved. Also, the elements of $\sum_{t=1}^{T}\sqrt{(\tilde{\mathbf{Z}}_{tj} - \mathbf{W}_{tj})^2/T} = (0.044,\ 0.101,\ 0.077)$ are very small, indicating that the preliminary time series are very close to the vector of final estimates. Considering $\mathbf{c}$, we obtain a weighted average of 0.084. On the other hand, with the Stuckey et al. (2004) approach the discrepan cies are also stationary and we obtain the following vector of root mean squared discrepancies of $\sum_{t=1}^{T}\sqrt{(\hat{\mathbf{W}}_{tj} - \mathbf{W}_{tj})^2/T} = (1.223,\ 0.019,\ 0.063)$, with a weighted average of 0.085. Consequently, our suggested approach yields, on average, slightly smaller differences between preliminary and final estimates, which is mainly due to the fact that the GA1 is considerably better adjusted.

Once we verified movement preservation, we now aim to guarantee the structural explanation between the aggregate and its disaggregates. Note that by definition $Y_t = \sum_{i=1}^{3} c_i \tilde{Z}_{it}$ which in turn implies $\Delta Y_t = \sum_{i=1}^{3} \Delta \tilde{Z}_{it}$. As we commented in the Introduction, in September 2006 all components of IGAE decreased but IGAE increased, and the opposite happened in July 2007.

Table 2 shows the monthly percentage changes for the preliminary estimates, final estimates and the reconciled estimates produced with the method of Stuckey et al. (2004).

We can see in Table 2 that in September 2006, IGAE decreased 0.03% and the preliminary estimates indicates that GA1, GA2 and GA3 increased by 2.02%, 0.05% and

Table 2. Monthly percentage changes for Y_t, $\mathbf{W}_t$, $\hat{\mathbf{Z}}_t$ and $\hat{\mathbf{W}}_t$ for IGAE, 2016–2017.

Date	Y_t	W_{1t}	W_{2t}	W_{3t}	$\tilde{Z}_{1t}$	$\tilde{Z}_{2t}$	$\tilde{Z}_{3t}$	$\hat{W}_{1t}$	$\hat{W}_{2t}$	$\hat{W}_{3t}$
2006/01	1.60	15.51	−0.54	1.35	15.84	0.04	1.88	24.74	−0.42	1.78
2006/02	−0.80	−8.62	−0.63	−0.09	−8.71	−0.87	−0.30	−11.33	−0.68	−0.25
2006/03	−0.05	−1.15	0.71	−0.25	−1.22	0.56	−0.38	−3.02	0.68	−0.36
2006/04	0.68	2.37	0.38	0.68	2.41	0.44	0.74	3.25	0.39	0.73
2006/05	1.63	2.21	0.92	1.41	2.41	1.33	1.78	7.66	1.00	1.71
2006/06	−1.10	−1.53	−0.27	−0.76	−1.78	−0.81	−1.24	−8.13	−0.38	−1.15
2006/07	0.21	−0.50	0.04	0.03	−0.40	0.26	0.22	2.36	0.08	0.19
2006/08	0.30	0.31	0.30	0.10	0.37	0.43	0.21	1.90	0.33	0.19
2006/09	**−0.03**	**2.02**	**0.05**	**0.12**	**1.92**	**−0.15**	**−0.06**	**−0.64**	**0.01**	**−0.03**
2006/10	0.44	2.40	−0.40	0.41	2.54	−0.10	0.68	6.42	−0.34	0.62
2006/11	−0.55	−4.95	0.18	−0.29	−5.08	−0.13	−0.56	−8.63	0.12	−0.51
2006/12	0.54	13.81	−0.41	0.45	13.82	−0.42	0.45	13.78	−0.41	0.45
2007/01	−0.05	−7.36	−0.43	0.23	−7.26	−0.18	0.45	−4.34	−0.38	0.40
2007/02	0.24	2.67	0.76	0.05	2.59	0.59	−0.10	0.42	0.73	−0.07
2007/03	0.24	0.44	0.73	0.39	0.30	0.43	0.12	−3.50	0.67	0.18
2007/04	0.59	−4.59	−0.19	1.00	−4.48	0.06	1.22	−1.42	−0.14	1.17
2007/05	0.53	1.68	−0.02	0.44	1.79	0.23	0.65	4.84	0.03	0.61
2007/06	0.24	4.41	0.58	0.46	4.21	0.15	0.08	−1.19	0.50	0.15
2007/07	**0.09**	**−2.93**	**−0.14**	**−0.12**	**−2.78**	**0.20**	**0.18**	**1.44**	**−0.08**	**0.12**
2007/08	−0.01	2.09	−1.36	0.71	2.10	−1.35	0.71	2.19	−1.36	0.71
2007/09	0.49	−2.25	1.61	0.14	−2.30	1.49	0.04	−3.69	1.59	0.06
2007/10	0.36	3.99	−0.33	0.11	4.14	−0.01	0.39	8.20	−0.27	0.33
2007/11	−0.75	3.70	−0.71	−0.36	3.50	−1.15	−0.74	−1.84	−0.79	−0.66
2007/12	0.00	−14.85	0.86	0.24	−14.83	0.92	0.30	−14.17	0.87	0.28

0.12% respectively, which is a very confusing result. Alternatively, our estimates indicate that this drop in the economy is attributed to decreases in GA2 and GA3, that is, the industrial and services sectors, although the primary sector increased. On the other hand, the estimates produced by the method of Stuckey et al. (2004) indicate that the decrease is attributed to decreases in GA1 and GA3.

In July of 2007, IGAE increased 0.09% but the preliminary estimates had growths of −2.93%, −0.14% and −0.12% respectively for the three GA. Similarly to the previous case, this growth in the economy is explained by positive growths in GA2 and GA3. The estimates provided by the method of Stuckey et al. (2004) point out that the growth of IGAE is due to positive growths in GA1 and GA3. However, since the weight of GA1 with respect to the total economy is modest (0.032), it is more plausible to attribute the movement of the whole economy to GA2 and GA3. In this sense, our proposal produces results that can be considered more reasonable, as can be seen in Figure 3.

This figure allows us to visually appreciate in particular how movement preservation is achieved. For GA1 the results obtained with the CR follow more closely those of the preliminary estimates, for GA2 the results obtained with Stuckey et al. (2004) are slightly better, and for GA3 are practically the same. Consequently, our suggested approach is an alternative to indirect seasonal adjustment, with the advantage that it can be validated empirically, since it is supported by statistical models. Once the empirical validation is achieved, we can claim statistical optimality of the results produced by our method.

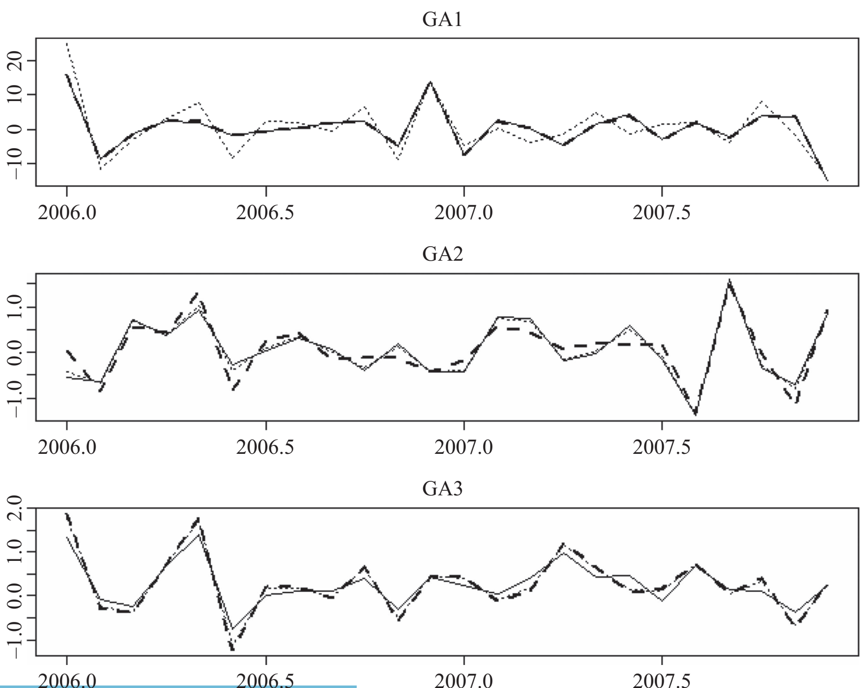

Fig. 3. Deseasonalized components of IGAE. Solid lines are the preliminary estimates ($\mathbf{W}_t$), dotted-dark lines are the series reconciled with the CR ($\hat{\mathbf{Z}}_t$) and the dotted-light lines are seasonally adjusted series with the procedure of Stuckey et al. (2004) ($\hat{\mathbf{W}}_t$).

6. Conclusions and Further Research

In this work, we propose to use a particular case of the CR of Guerrero and Peña (2000, 2003), exposited in Guerrero and Nieto (1999) to reconcile the disaggregates obtained as directly deseasonalized time series, produced by X-13ARIMA-SEATS, with its respective aggregate. The suggested approach is based on estimating a VAR(p) model for the preliminary estimates (disaggregates), on the assumption that they follow the same model as the vector to be restricted.

If the discrepancies between the reconciled time series and the preliminary estimates are White Noises, the model estimation is finished; otherwise, a new VAR(p) model for the discrepancies has to be estimated in order to consistently estimate the final restricted time series. Then, we empirically verify movement preservation between the reconciled time series and the preliminary estimates.

We evaluate the finite sample performance of the proposed procedure by means of a Monte Carlo study that considers six DGPs that simulate different situations that may occur when dealing with correlated seasonal time series. Specifically, we try to simulate situations faced by INEGI's personnel in charge of seasonal adjustment. We conclude that the suggested approach allows us to jointly attain structural explanation and movement preservation in several cases. For instance, with respect to the observed discrepancy between the disaggregates of IGAE with its aggregate in September 2006 and July 2007, we verified that the suggested procedure adequately reconciles the preliminary estimates.

This approach can be applied to different banks of official information, since the procedure is supported by just a few assumptions to guarantee its optimality from a statistical point of view; additionally, it is easy to implement computationally. In contrast, its main disadvantage is that the suggested approach relies on estimating several parameters. Consequently, this method is feasible only when the sample size is much larger than the number of disaggregate time series.

In fact, the proposed method works well with VAR models of moderate dimension, no more than around ten disaggregated series. In that case, the number of parameters for the ten equations involved in a VAR(p) model will require estimating $p \times 100$ regression parameters. Thus, the explosion of parameters in situations where the sample size is moderate (around 120 observations), will render the method infeasible or, at least, inaccurate. For that reason, we should bear in mind that sparse modelling may be needed for large dimensional situations (see Hsu et al. 2008; Davis et al. 2016; and Wilms et al. 2017). Another possibility could be to use a Bayesian VAR (BVAR) model with for instance, a Minnesota prior, as in Doan et al. (1984) and (Lütkepohl, 2005 chap. 5).

It should also be clear that our proposal does not provide an easy-to-use method that could be applied by everyone working at a statistical agency. Its application requires familiarity with formal statistical model building, which we consider indispensable nowadays for people in charge of applying seasonal adjustment techniques professionally.

Further research related to the Monte Carlo experiments is required, so that we should consider more hierarchical structures in sectoral time series, different number of time series and observations of those time series and common seasonal patterns. Also, it would be advisable to take into account different types of serial correlation in the idiosyncratic errors. It is also advisable to study the non-stationary, but non-cointegrated case, which theoretically requires to estimate a VAR(p) model in first differences.

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